Numerical Integration Notes
Section 4.6

Some functions DON’T have antiderivatives such as \((\frac{3}{\sqrt{x}})(\sqrt{1-x})\), \((\sqrt{x})(\cos x), \frac{\cos x}{x}, \sqrt{3^2 - x^3}, \sqrt{1 + \sin^2 x}\) and \(\sin(x^2)\) so we need a better area approximation tool then rectangles…..

Call in the TRAPEZOIDs! Do you remember how to find the area of one? Sketch a trapezoid, label the dimensions with the appropriate variables and write your area formula for it in the space below.

We are going to look at dividing the area under a curve into trapezoids. Copy the notes your teacher gives you onto the diagram below.

Graph of \(f(x) = (\sqrt{x})(3\sin x)\)
THEOREM 4.17 The Trapezoidal Rule

Let $f$ be ________________________________________________________________
________________________________________________________________________

\[
\int_{a}^{b} f(x) dx \approx \frac{b-a}{2n} \left[ f(x_0) + 2 f(x_1) + 2 f(x_2) + \ldots + 2 f(x_{n-1}) + f(x_n) \right]
\]

Moreover, as $n \to \infty$, the right-hand side approaches $\int_{a}^{b} f(x) dx$

We're going to do the first problem from today's assignment together....

Use the trapezoidal rule with $n = 4$ to approximate $\int_{0}^{2} \sqrt{1 + x^3} \, dx$. 
Example: Use four trapezoids to find the trapezoidal sum for
\[ \int_{1}^{12} g(x) \, dx \], where some values for \( g(x) \) are as given in the following table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>7</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td>4</td>
<td>6</td>
<td>10</td>
<td>16</td>
<td>20</td>
</tr>
</tbody>
</table>

It is also important to learn how to use the calculator to perform “impossible” integrals. To do this, follow these steps.

1. Enter \( f(x) \) (in this case, \( \sqrt{1+x^3} \)) into Y = .
3. Put in the lower limit (0), then press \( \ddot{\text{V}} \).
4. Put in the upper limit (2), then press \( \ddot{\text{V}} \).
5. To retrieve \( Y_1 \) without having to retype it, press ALPHA, then press TRACE. This will call up the Y= variables. Press ENTER to select \( Y_1 \), then press \( \ddot{\text{V}} \).
6. Enter \( x \) for \( dx \), then press ENTER.