The next important rules of derivatives are the derivatives of the sine and cosine. Remember the looong definition of derivative? Well, we’re going to use it to find the derivative of sine and cosine and from then on, we’ll use those results to MEMORIZE (you didn’t really want to use the definition of derivative all the time did you?) the derivatives of sine and cosine.

Okay, so let’s start with the definition of derivative:

\[ f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}. \]

So, let \( f(x) = \sin x \). Now, using the definition of the derivative above and \( \lim_{x \to 0} \frac{\sin x}{x} = 1 \) and \( \lim_{x \to 0} \frac{1 - \cos x}{x} = 0 \), find the derivative of \( \sin x \). Show all steps.
Basic Differentiation Rules and Rates of Change Notes
Section 2.2b

Now, let $f(x) = \cos x$. Using the definition of derivative and
$$\lim_{x \to 0} \frac{\sin x}{x} = ____ \quad \text{and} \quad \lim_{x \to 0} \frac{1 - \cos x}{x} = ____,$$
find the derivative of $\cos x$. Show all steps.

Now that you've learned the derivatives of the sine and cosine, try the problems below.

1. $f(x) = \sin x - 3x^2$

2. $g(x) = 7x^2 + 2\cos x - 5$

3. $h(x) = \frac{1}{5} \sin x - \frac{\cos x}{6} + 6x^3$