Read the Differentiability and Continuity section on page 101 and fill in the missing information below.

The following alternative limit form of the derivative is useful in investigating this relationship. The derivative of $f$ at $c$ is given by

$$f'(c) = \text{__________________________}$$

Alternate form of derivative

provided this limit exists. Note that the existence of the limit in this alternative form requires that the one-sided limits

$$\lim_{x \to c^-} \text{____________________} \quad \text{and} \quad \lim_{x \to c^+} \text{____________________}$$

exist and are equal. These one-sided limits are called the ____________ ____________________________________________________________________________, respectively. We say that $f$ is ____________________________________________________________________________

______________________________________________________________________________

Study the examples on page 102. Note that they use the ALTERNATE FORM OF THE DERIVATIVE which is also called the derivatives from the left and from the right.

Copy and title the graph in figure 2.11 below.
The greatest integer function on the top of page 101 illustrates this important point: if a function is not continuous at \( x = c \), then it is also not differentiable at \( x = c \). Can you draw a tangent line at \( x = 0 \) for \( f(x) = \lfloor x \rfloor \)? _____ Use complete sentences to explain why or why not. ________________________________________________________________

Although it is true that differentiability implies continuity, the converse is NOT true. That is, it IS possible for a function to be continuous at \( x = c \) and NOT differentiable at \( x = c \). Examples 6 and 7 illustrate this possibility.

Copy and title the graph of the function in Example 6 below.

\[ y \]
\[ x \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \]

Is the function in Example 6 continuous? _____
Can you draw a tangent line at \( x = 2 \)? _____
Use complete sentences to explain why or why not. __________________________

Use the Alternate Form of the Derivative to prove that the derivative does not exist at \( x = 2 \) for \( f(x) = |x - 2| \).
The Derivative and Tangent Line Problem Notes
Section 2.1b

Copy and title the graph of the function in Example 7 here.

Is the function in Example 7 continuous? ___
Can you draw a tangent line at $x = 0$? ____
Use complete sentences to explain why or why not.
__________________________________________________________
__________________________________________________________
__________________________________________________________

Use the Alternate Form of the Derivative to prove that the derivative does not exist at $x = 0$ for $f(x) = \frac{1}{x^3}$.

From Examples 6 and 7, you can see that a function is not differentiable at a point at which its graph has a ____________________________
__________________________________________________________________________

**Theorem 2.1 Differentiability Implies Continuity**

If $f$ is differentiable at $x = c$, then $f$ ________________________________

You can summarize the relationship between continuity and differentiability as follows.
The Derivative and Tangent Line Problem Notes  
Section 2.1b

1. If a function is differentiable at $x = c$, ________________________________
   Thus, differentiability __________________________________________________

2. It is possible for a function to be continuous at $x = c$ and not _____
   _______________ Thus, continuity does not __________________________

Consider the function $f(x) = \begin{cases} 
\frac{3}{2}x - \frac{1}{2} & \text{for } x < 3 \\
2 & \text{for } x \geq 3 \\
x^2 - 5 & \text{for } x \geq 3 
\end{cases}$.

1. First, check for continuity:

2. Use the Alternate Form of the Derivative to test to see if the derivative from the left equals the derivative from the right.
3. State your conclusion in a complete sentence.

_____________________________________________________________________

_____________________________________________________________________

_____________________________________________________________________