Finding the Slope of a Tangent Line and Discovering the Derivative

Find the slope of the tangent line at (1, 1).

Looking at the table we created, what would you guess is the slope of the line tangent to the graph of \( y = x^2 \) at the point (1, 1)? ________

<table>
<thead>
<tr>
<th>( x_2 )</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1.5</th>
<th>1.2</th>
<th>1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work on Slope</td>
<td></td>
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</tr>
<tr>
<td>slope</td>
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</tbody>
</table>

Looking at the table we created, what would you guess is the slope of the line tangent to the graph of \( y = x^2 \) at the point (1, 1)? ________

The beauty of this procedure is that you can obtain more and more accurate approximations to the slope of the tangent line by ________

______________________________________________________________
The Derivative of a Function

You have now arrived at a crucial point in the study of calculus. The limit used to define the slope of a tangent line is also used to define one of the two fundamental operations of calculus — differentiation.

Note that the definition of the Derivative of a Function is EXACTLY the same as the Definition of the Tangent Line with Slope m (the $c$ has just been changed to $x$).

**Definition of the Derivative of a Function**

The derivative of $f$ at $x$ is given by

$$f'(x) = \frac{dy}{dx} = \lim_{{\Delta x \to 0}} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

provided the limit exists.

Notice that there are 5 different notations used to denote the derivative of a function:

$$f'(x), \quad \frac{dy}{dx}, \quad y', \quad \frac{d}{dx} [f(x)], \quad D_x [y].$$

We’ll mainly be using the first 3 forms.

Study the examples on pages 99-100 and then try the problems on the next page.
Using the definition of derivative, let’s find the derivative for our function, \( f(x) = x^2 \).

Using the definition of derivative, find the derivative, \( f'(x) \).

1. \( f(x) = 5x^2 - 2x \)
2. Find the equation of the tangent line to \( f(x) = 5x^2 - 2x \) at the point \((1, 3)\).