There are three main types of graphs you should know how to graph by hand. They are _________, __________________, and ____________________.

**Petal Graphs**

Petal graphs are in the form $r =$ ____________ or ______________ where _______. The maximum radius for any petal is $a$. In cosine graphs, the first peak is always at $\theta = 0\pi$. In sine graphs the first peak is always at __________. In both cases, there are $n$ petals if $n$ is odd and $2n$ petals if $n$ is even.

Example 1: $r = 2\cos(3\theta)$. The maximum radius is ___ and the number of petals is _____. The first peak is at ______.

To graph a petal graph, lightly sketch a circle of radius $a$. Locate its ______________. From the ______________, divide the circle in $n$ parts if $n$ is odd and $2n$ parts if $n$ is even. In this problem, we need to divide the circle into ___ parts. Mark off points that correspond to the number of petals.

Determine the order of the peaks. The second peak must be at least $\pi$ radians away from the first peak.

Beginning at $\theta = 0$, draw a curve through the pole and to the $2^{nd}$ peak.

Then draw a curve through the pole to the $3^{rd}$ peak.
Finally, draw the last branch of the curve through the pole to the beginning.

Example 2: $r = 3\sin(2\theta)$

We know that the maximum radius is ____ and the number of petals is ____ = ____. The first peak is at $\theta = \frac{\pi}{2n} = \frac{\pi}{4}$. Draw this point on your sketch. Since we need ____ petals, divide the rest of the circle into __________.

The other peaks will occur at ____, ____ and _____. Determine the order of the peaks. (Remember: draw curves that pass through the origin and are one peak further around the circle than $\pi$ radians).
Polar Coordinates
Section 10.4b

Circles
The polar equation for a circle is ______________ or ______________ or __________.
If the polar equation is of the first 2 types, graph them like a petal graph (but the petal is a circle).
Example 1: \( r = 2 \cos \theta \).

Limacons
These are equations in the form ______________ or ______________. Use the following ideas to sketch the graph:

1. **Orientation** - If the function involves cosine, the graph will be positioned ______________. If the function involves sine, the graph will be positioned ______________.

2. **Intercepts** – Use the quadrantal angles \( 0\pi, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \) to find the intercepts of the x and y axes.

3. **Shapes** – The shapes depend on the relationship between \( |a| \) and \( |b| \).

   A. If ____________, then the function is a dimpled limaçon:

   B. If ____________, then the function is a cardioid:

   C. If ____________, then the function is a looped limaçon:

   D. If ____________, then the function is a convex limaçon:
Example 1: \( r = 2 - 2\sin\theta \).

1. Determine the shape – ______________________________________________________________________

2. Find the intercepts: Use \( \theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \).

3. Graph the limaçon.

Example 2: \( r = 2 - 3\cos\theta \)

1. Determine the shape – ______________________________________________________________________

2. Find the intercepts: Use \( \theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \).

3. Graph the limaçon.
Lemniscates:
Lemniscates are in the form \[ r^2 = a^2 \cos(2\theta) \] or \[ r^2 = a^2 \sin(2\theta) \]. They always look like figure eights. Cosine lemniscates are always oriented along the positive and negative \[ \theta \]. Sine lemniscates are always oriented along the line \[ \theta = \frac{\pi}{4} \]. Maximum distance out is \[ a \].

Example: \[ r^2 = 3\sin(2\theta) \]. This will be oriented along the \[ \theta = \frac{\pi}{4} \]. Maximum distance from the pole is \[ 3 \]. The shape is a figure 8.