Infinite Limits Notes  
Section 1.5

Read page 83, skip the box on Definition of Infinite Limits and fill in the missing information below.

Let $f$ be the function given by $f(x) = \frac{3}{x-2}$. From Figure 1.39 and the table, you can see that $f(x)$ __________________________________________
______________________________________________________________________________
______________________________________________________________________________
This behavior is denoted as $\lim_{x \to 2^-} \frac{3}{x-2} = \text{______}$ and $\lim_{x \to 2^+} \frac{3}{x-2} = \text{______}$.

Complete the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1.5</th>
<th>1.9</th>
<th>1.99</th>
<th>1.999</th>
<th>2</th>
<th>2.001</th>
<th>2.01</th>
<th>2.1</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$f(x)$ decreases _______________ $f(x)$ increases _______________

Copy Figure 1.39 below.

$f(x)$ _______________________________ as $x$ approaches 2.

A limit in which $f(x)$ increases or decreases without bound as $x$ approaches $c$ is called ____________________________
Look at the bottom of page 83 and complete these sentences.

Be sure you see that the equal sign in the statement
\[ \lim_{x \to \_} f(x) = \infty \]
On the contrary, it tells you how the limit ____________________
______________________________
______________________________

Read and study example 1 on page 84 and then try the problems below.

Graph each function. For each function, analytically find the single real number c that is not in the domain. Then graphically find the limit of \( f(x) \) as \( x \) approaches \( c \) from the left AND from the right. Sketch and title the graph of each function.

a. \( f(x) = \frac{3}{x-4} \quad x \neq \_ \)

\[ \lim_{x \to -} \frac{3}{x-4} = \_ \]

\[ \lim_{x \to +} \frac{3}{x-4} = \_ \]

b. \( f(x) = \frac{3}{2-x} \quad x \neq \_ \)

\[ \lim_{x \to -} \frac{3}{2-x} = \_ \]

\[ \lim_{x \to +} \frac{3}{2-x} = \_ \]
c. \( f(x) = \frac{2}{(x-3)^2} \) \( x \neq ____ \)

\[
\lim_{x \to -} \frac{2}{(x-3)^2} = ____.
\]

\[
\lim_{x \to +} \frac{2}{(x-3)^2} = ____.
\]

d. \( f(x) = \frac{-3}{(x+2)^2} \) \( x \neq ____ \)

\[
\lim_{x \to -} \frac{-3}{(x+2)^2} = ____.
\]

\[
\lim_{x \to +} \frac{-3}{(x+2)^2} = ____.
\]

Skim pages 85-86 (it should be review) and copy the information in the box on page 87 on the next page. Skip the proof but read and study example 5 and fill in the missing information.
THEOREM 1.15 Properties of Infinite Limits

Let c and L be real numbers and let f and g be functions such that
\[ \lim_{x \to c} f(x) = \infty \quad \text{and} \quad \lim_{x \to c} g(x) = L. \]

1. Sum or difference:
   \[ \infty + L = \infty \]

2. Product:
   \[ \infty \cdot L = \infty \]

3. Quotient:
   \[ \frac{\infty}{L} = \infty \]

Similar properties hold for one-sided limits and for functions for which the limit of \( f(x) \) as \( x \) approaches \( c \) is \(-\infty\).

EXAMPLE 5 Determining Limits

a. Because the \( \lim_{x \to 0} 1 = 1 \) and \( \lim_{x \to 0} \frac{1}{x^2} = \infty \), you can write

\[
\lim_{x \to 0} \left( 1 + \frac{1}{x^2} \right) = \infty + 1
\]

\[ = \infty + 1 \]

\[ = \infty, \quad \text{Property 1, Theorem 1.15} \]
b. Because the \( \lim_{x \to 1^-}(x^2 + 1) = \) ___ and \( \lim_{x \to 1^-} (\cot \pi x) = \) ___, you can write
\[
\lim_{x \to 1^-} \frac{x^2 + 1}{\cot \pi x} = \underline{\text{______________}}
\]
\[
= \underline{\text{______________}}
\]
\[
= \underline{\text{____}} \quad \text{Property 3, Theorem 1.15}
\]

c. Because the \( \lim_{x \to 0^+} 3 = \) ____ and \( \lim_{x \to 0^+} \cot x = \) ___, you can write
\[
\lim_{x \to 0^+} 3 \cot x = \underline{\text{______________}}
\]
\[
= \underline{\text{______________}}
\]
\[
= \underline{\text{____}} \quad \text{Property 2, Theorem 1.15}
\]

d. Because the \( \lim_{x \to 0^-} x^2 = \) ____ and \( \lim_{x \to 0^-} \frac{1}{x} = \) ___, you can write
\[
\lim_{x \to 0^-} \left( x^2 + \frac{1}{x} \right) = \underline{\text{______________}}
\]
\[
= \underline{\text{______________}}
\]
\[
= \underline{\text{____}} \quad \text{Property 1, Theorem 1.15}
\]