Read pages 72 – 73 and fill in the missing information below.

\[ \lim_{{x \to c^+}} f(x) = L. \]

The limit ______________________________________ _________
________________________________________________________________________

\[ \lim_{{x \to c^-}} f(x) = L. \]

The limit ______________________________________ _________
________________________________________________________________________

EXAMPLE 2 A One-Sided Limit

Find the limit of \( f(x) = \sqrt{4 - x^2} \) as \( x \) approaches \(-2\) from the right.

In limit notation, this would be written as ______________________________

Fill in the missing values in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2)</th>
<th>(-1.9999)</th>
<th>(-1.99)</th>
<th>(-1.9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What value is \( f(x) \) approaching as \( x \) goes from \(-1.9\) to \(-2\)? _______

Copy the graph from figure 1.29 here.

What does \( f(x) \) (the y-value) approach as you trace the graph from the right to \( x = -2 \) with your finger? _________________
________________________________________________________________________

So, the \( \lim_{{x \to -2^+}} \sqrt{4 - x^2} = \) _____.

What does the \( \lim_{{x \to -2^-}} \sqrt{4 - x^2} = \) _____?
EXAMPLE 3 The Greatest Integer Function

Fill in the missing values in the table below for \( f(x) = \lfloor x \rfloor \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-.1)</th>
<th>(-.01)</th>
<th>(-.001)</th>
<th>(-.0001)</th>
<th>( 0 )</th>
<th>( .0001 )</th>
<th>( .001 )</th>
<th>( .01 )</th>
<th>( .1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Copy the graph from figure 1.27 here.

\[
\lim_{x \to 0^-} \lfloor x \rfloor = \text{______}
\]

\[
\lim_{x \to 0^+} \lfloor x \rfloor = \text{______}
\]

When the limit from the left is not equal to the limit from the right, the (two-sided) limit does not exist. The next theorem (on page 73) makes this more explicit.

**Theorem 1.10 The Existence of a Limit**

Let \( f \) be a function, and let \( c \) and \( L \) be real numbers. The limit of \( f(x) \) as \( x \) approaches \( c \) is \( L \) if and only if

From example 3 above, because \( \lim_{x \to 0^-} \lfloor x \rfloor = \text{______} \) and \( \lim_{x \to 0^+} \lfloor x \rfloor = \text{______} \)

therefore the \( \lim_{x \to 0} \lfloor x \rfloor = \text{______} \).
Definition of Continuity

Continuity at a Point: A function \( f \) is ______________________________________

______________________________________________________________________________

1. \( f(c) \) is _________________
2. \( \lim_{x \to c} f(x) \) _______________
3. \( \lim_{x \to c} f(x) = \) _________

Continuity on an Open Interval: __________________________________________

______________________________________________________________________________

Use the definition of continuity to decide if \( f(x) = \begin{cases} 
\frac{x+2}{2}, & x \leq 3 \\
\frac{12x-2x}{3}, & x > 3 
\end{cases} \) is continuous.