Evaluating Limits Analytically

Section 1.3b

In section 1.1, you used tables and graphs to find the limits of functions at “trouble spots” in their domains. In section 1.2, you used direct substitution as a shortcut to find limits of functions at “non-trouble spots” in their domains. We want to be able to use direct substitution for finding limits of functions at “trouble spots” in their domains, so in this section we will learn some strategies that will allow us to do that with *some* functions.

First we must understand what the term, *indeterminate form* of an expression means. If we use direct substitution and get fractions in the form of: \( \frac{0}{0} \) or \( \frac{\pm\infty}{\pm\infty} \), these results are called indeterminate forms (there are other indeterminate forms but they are dealt with in the AP Calculus BC course) because we cannot determine the limit of expressions in this form.

**Cancellation Technique**

Try using direct substitution for \( \lim_{x \to 1} \frac{x^3 - 1}{x - 1} \).

\[
\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \frac{\text{___________________________}}{\text{___________________________}} = \text{_______} = \text{__________}
\]

We cannot say what this number is, since we can’t divide by zero. So, instead, we factor the numerator, simplify and THEN use direct substitution.

\[
\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \frac{\text{___________________________}}{\text{___________________________}} = \text{___________________________}
\]

\[
= \text{___________________________}
\]

\[
= \text{___________________________}
\]

= _____

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Now, you try some. Show all steps.

1. \( \lim_{x \to -3} \frac{x^2 + x - 6}{x + 3} \)

2. \( \lim_{x \to 4} \frac{x^2 - 16}{x - 4} \)

3. \( \lim_{x \to -3} \frac{x^3 + 27}{x + 3} \)

4. \( \lim_{x \to 3} \frac{7x^2 - 19x - 6}{x - 3} \)

Rationalization Technique

Try using direct substitution for \( \lim_{x \to 0} \frac{\sqrt{x + 1} - 1}{x} \).

\[
\lim_{x \to 0} \frac{\sqrt{x + 1} - 1}{x} = \text{__________________________}
\]

\[
= \text{____________}
\]

\[
= \text{______}
\]

\[
= \text{__________}
\]

We cannot say what this number is, since we can’t divide by zero.
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So, we’re going to rid of the zero in the denominator by rationalizing (getting rid of the square root in the numerator) this fraction by multiplying by 1. But the number 1 looks like this:

\[ \frac{\sqrt{x+1} - 1}{x} \]

So, we have

\[ \lim_{x \to 0} \left( \frac{\sqrt{x+1} - 1}{x} \right) \left( \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1} \right) = \ldots \]

Okay, now you try some. Show all steps.

1. \[ \lim_{x \to 0} \left( \frac{\sqrt{4 + x} - 2}{x} \right) \]

2. \[ \lim_{x \to 4} \left( \frac{\sqrt{x} - 2}{x - 4} \right) \]