Evaluating Limits Analytically
Section 1.3a

In the last section, we found limits at “trouble spots” of the function. In this section, we will find an easier way to find limits BUT ONLY AT CONTINUOUS SPOTS OF THE FUNCTION.

Sketch and title the graph of \( f(x) = 3 \) below.

\[ y \]
\[ x \]

a. Is it continuous everywhere? ______
b. Is it continuous at \( x = 2 \)? _______
c. \( \lim_{x \to 2^-} 3 = _____ \)
d. \( \lim_{x \to 2^+} 3 = _____ \)
e. \( \lim_{x \to 2} 3 = _____ \)
f. \( \lim_{x \to 2} 3 = _____ \)
g. \( \lim_{x \to 4} 3 = _____ \)
h. \( \lim_{x \to 130} 3 = _____ \)

This is an example of the Constant Rule strategy for finding limits. Try these problems.

i. \( \lim_{x \to 7} 152 = _____ \)

j. \( \lim_{x \to 20} 1534 = _____ \)

k. \( \lim_{x \to -58} (-678) = _____ \)
Evaluating Limits Analytically
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Sketch and title the graph of \( f(x) = x \) below.

- a. Is it continuous everywhere? _____
- b. Is it continuous at \( x = -4 \)? ______
- c. \( \lim_{x \to -4^-} x = ____ \)
- d. \( \lim_{x \to -4^+} x = ____ \)
- e. \( \lim_{x \to -4} x = ____ \)

Sketch and title the graph of \( f(x) = x^3 \) below.

- a. Is it continuous everywhere? _____
- b. Is it continuous at \( x = -2 \)? ______
- c. \( \lim_{x \to -2^-} x^3 = ____ \)
- d. \( \lim_{x \to -2^+} x^3 = ____ \)
- e. \( \lim_{x \to -2} x^3 = ____ \)

This is an example of the Direct Substitution strategy for finding limits.
Try these problems.

1. \( \lim_{x \to 2} 4x^2 + 3 = \)

2. \( \lim_{x \to 5} (3x^2 - 4x + 1) = \)

3. \( \lim_{x \to -1} (5x + 2) = \)

The Limit of a Rational Function

Find the limit: \( \lim_{x \to 1} \frac{x^2 + x + 2}{x + 1} = \)

\( \lim_{x \to 1} \frac{x^2 + x + 2}{x + 1} = \) _____ = ____ = ____
WHY CAN’T YOU USE THE DIRECT SUBSTITUTION PROPERTY TO FIND $\lim_{x \to -1} \frac{x^2 + x + 2}{x + 1}$? ________________________________

Use a graphing calculator to look at the graph of the function above. Copy that graph on the graph below. Make sure to title the graph and label any holes and/or asymptotes.

Using the graph, find the $\lim_{x \to -1} \frac{x^2 + x + 2}{x + 1} = ___$

Read and study the remaining theorems and examples on pages 60-61.

REMEMBER, WE CAN ONLY USE DIRECT SUBSTITUTION TO FIND LIMITS AT POINTS ON THE FUNCTION WHERE THE FUNCTION IS CONTINUOUS!!!!

That means that we have to be careful about finding limits on rational, radical tangent, cotangent, secant and cosecant functions.