Limits are important because they can help us find the y-value at questionable spots in the domain of a function. Understanding limits is crucial to understanding calculus because limits lead right in to derivatives – a major part of calculus.

Limits can either:

1. exist or
2. not exist (abbreviated D.N.E. – Does Not Exist).

We can find limits:

1. numerically by using a table, or
2. graphically, or
3. analytically.

Today, we will explore how to find limits numerically by using a table and by analyzing a graph. Over the next couple of days, we will also learn how to find limits analytically.

In Pre-Cal and in the Skill Review, you reviewed rational functions. This is one type of function that has domain restrictions. It is important to understand what is happening at the restricted part of the domain. That is where limits come in handy.

Let’s take a look at a problem.

1. For the function \( f(x) = \frac{x^2 - 1}{x - 1} \):
   a) factor each expression in the function. Write the factored function here:
   
   \( f(x) = \) __________
Section 1.2a Finding Limits Numerically and Graphically

b) identify holes and vertical asymptotes.

Hole at ( , ).

Vertical asymptote at \( x = \) _____.

c) identify the horizontal asymptotes.

Horizontal asymptote at \( y = \) _____.

d) graph the function. Label the hole(s) and the asymptote(s) and title the graph.

![Graph with grid and axes](image)

e) Fill in the following table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>.9</th>
<th>.99</th>
<th>.999</th>
<th>1</th>
<th>1.001</th>
<th>1.01</th>
<th>1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td>?</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

f) What does \( f(x) \) do as you follow the arrow from the left towards 1? Write your observation by filling in the following approach statement. As \( x \) approaches ___ from the left, \( f(x) \) approaches ___.

2
Section 1.2a Finding Limits Numerically and Graphically

g) What does \( f(x) \) do as you follow the arrow from the right towards 1? Write your observation here using an approach statement. ____________________________________________________________________________
________________________________________________________________________

h) Go back and look at your graph in part d. Put your left index finger on the furthest point on the left of the graph of \( f(x) \) and put your right index finger on the furthest point on the right of the graph of \( f(x) \). Following the shape of the graph, move your fingers toward each other and towards \( f(1) \).

What \( y \)-value does your left index finger approach as you move towards \( f(1) \)? Write your answer here using an approach statement.

What \( y \)-value does your right index finger approach as you move towards \( f(1) \)? Write your answer here using an approach statement.

i) Compare your answers to parts f, g and h, then fill in the information below.

\[
\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \_\_.
\]
Section 1.2a Finding Limits Numerically and Graphically

2. For the function, \( f(x) = \frac{1}{x - 2} \):

   a) factor each expression in the function. Write the factored function here:

   \[ f(x) = \ldots \]

   b) identify holes and vertical asymptotes.

   Hole at ( , ).

   Vertical asymptote at \( x = \ldots \).

   c) identify the horizontal asymptotes.

   Horizontal asymptote at \( y = \ldots \).

   d) graph the function. Label the hole(s) and the asymptote(s) and title the graph.

   ![Graph of the function]

   e) Fill in the following table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.9</th>
<th>1.99</th>
<th>1.999</th>
<th>2</th>
<th>2.001</th>
<th>2.01</th>
<th>2.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td>?</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section 1.2a Finding Limits Numerically and Graphically

f) What does \( f(x) \) do as you follow the table and the graph from the left towards 2? Write your observation here using an approach statement.

g) What does \( f(x) \) do as you follow the table and the graph from the right towards 2? Write your observation here using an approach statement.

h) Compare your answers to parts f and g, then fill in the information below.

The \( \lim_{x \to 2} \frac{1}{x - 2} = _____ \).

3. For the function \( f(x) = \frac{|x|}{x} \):

a) rewrite the function as a piecewise function.

\[
f(x) = \begin{cases} \frac{x}{x} & \text{if } x \geq ____ \\ -\left(\frac{x}{x}\right) & \text{if } x \leq ____ \\ \end{cases}
\]

b) simplify the piecewise function above.

\[
f(x) = \begin{cases} \text{______} & \text{if } x \geq ____ \\ \text{______} & \text{if } x \leq ____ \\ \end{cases}
\]
Section 1.2a Finding Limits Numerically and Graphically

c) identify holes and vertical asymptotes.

Hole at ( , ).

Vertical asymptote at $x = ____$. 

d) identify the horizontal asymptotes.

Horizontal asymptote at $y = ____$. 

e) graph the function. Label the hole(s) and the asymptote(s) and title the graph.

f) Fill in the following table. What numbers should you choose for $x$ and why?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>?</th>
</tr>
</thead>
</table>


g) What does $f(x)$ do as you follow the table and graph from the left towards $x = ___$? Write your observation here using an approach statement.

h) What does $f(x)$ do as you follow the table and graph from the right towards $x = ___$? Write your observation here using an approach statement.
Section 1.2a Finding Limits Numerically and Graphically

i) Compare your answers to parts g and h, then fill in the information below.

The \( \lim_{x \to \_} \frac{|x|}{x} = \_ \).

4. Consider the piecewise function

\[ f(x) = \begin{cases} 1, & x \neq 2 \\ 0, & x = 2 \end{cases} \]

a) Graph and title the function.

b) Build a table for finding the limit of the function. What values should you choose for \( x \) and why?

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What does \( f(x) \) do as you follow the table and graph from the left towards \( x = \_ \)? Write your observation here using an approach statement.

d) What does \( f(x) \) do as you follow the table and graph from the right towards \( x = \_ \)? Write your observation here using an approach statement.
Section 1.2a Finding Limits Numerically and Graphically

e) Compare your answers to parts c and d, then fill in the information below.

The \( \lim_{x \to 2} f(x) = \begin{cases} 
1, & x \neq 2 \\
0, & x = 2
\end{cases} \).  

You’ve now looked at two piecewise functions. One had a limit that existed and one had a limit that did not exist. Why did one exist and the other did not? Write your observation here using complete sentences. Your answer should use approach statements.

__________________________________________________________________________________
__________________________________________________________________________________
__________________________________________________________________________________
__________________________________________________________________________________
__________________________________________________________________________________
__________________________________________________________________________________

8